1.5 Segment and Angle Bisectors

What *you should learn*

GOAL 1 Bisect a segment.

GOAL 2 Bisect an angle, as applied in **Exs. 50–55**.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding the angle measures of a kite in **Example 4**.

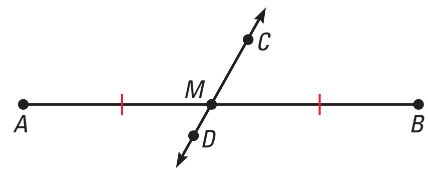
REAL LIFE

GOAL 1 BISECTING A SEGMENT

The **midpoint** of a segment is the point that divides, or **bisects**, the segment into two congruent segments. In this book, matching red *congruence marks* identify congruent segments in diagrams.

A **segment bisector** is a segment, ray, line, or plane that intersects a segment at its midpoint.

***M* is the midpoint of $\overline{AB}$ if *M* is on $\overline{AB}$ and *AM* = *MB*.**

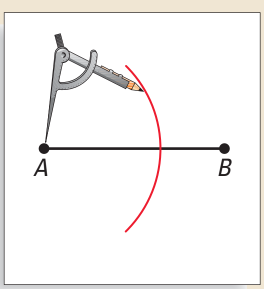


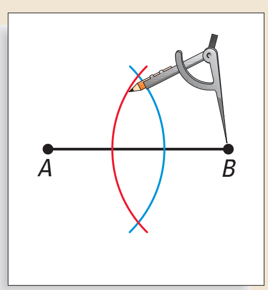
$\overleftrightarrow{CD}$ is a bisector of $\overline{AB}$.

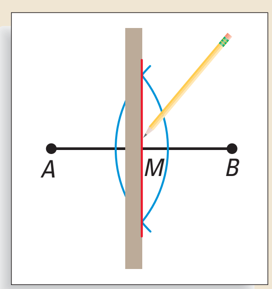
You can use a **compass** and a **straightedge** (a ruler without marks) to **construct** a segment bisector and midpoint of $\overline{AB}$. A **construction** is a geometric drawing that uses a limited set of tools, usually a compass and a straightedge.

ACTIVITY Construction — Segment Bisector and Midpoint

Use the following steps to construct a bisector of $\overline{AB}$ and find the midpoint *M* of $\overline{AB}$.







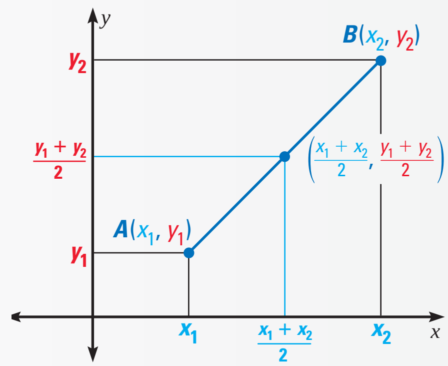
1. Place the compass point at *A*. Use a compass setting greater than half the length of $\overline{AB}$. Draw an arc.
2. Keep the same compass setting. Place the compass point at *B*. Draw an arc. It should intersect the other arc in two places.
3. Use a straightedge to draw a segment through the points of intersection. This segment bisects $\overline{AB}$ at *M*, the midpoint of $\overline{AB}$.

If you know the coordinates of the endpoints of a segment, you can calculate the coordinates of the midpoint. You simply take the mean, or average, of the x-coordinates and of the y-coordinates. This method is summarized as the **Midpoint Formula**.

THE MIDPOINT FORMULA

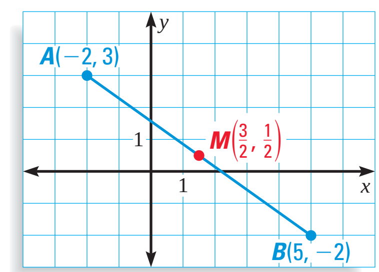
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



EXAMPLE 1 *Finding the Coordinates of the Midpoint of a Segment*

Find the coordinates of the midpoint of $\overline{AB}$ with endpoints $A(-2, 3)$ and $B(5, -2)$.



SOLUTION

Use the Midpoint Formula as follows.

$$M = \left(\frac{-2 + 5}{2}, \frac{3 + (-2)}{2}\right)$$

$$= \left(\frac{3}{2}, \frac{1}{2}\right)$$

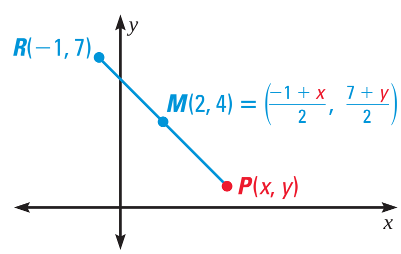
EXAMPLE 2 *Finding the Coordinates of an Endpoint of a Segment*

Using Algebra

The midpoint of $\overline{RP}$ is $M(2, 4)$. One endpoint is $R(-1, 7)$. Find the coordinates of the other endpoint.

SOLUTION

Let (x, y) be the coordinates of P. Use the Midpoint Formula to write equations involving x and y.



$$\frac{-1 + x}{2} = 2 \qquad \frac{7 + y}{2} = 4$$

$$-1 + x = 4 \qquad 7 + y = 8$$

$$x = 5 \qquad y = 1$$

▶ So, the other endpoint of the segment is $P(5, 1)$.

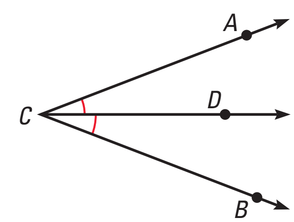
STUDENT HELP

Study Tip
Sketching the points in a coordinate plane helps you check your work. You should sketch a drawing of a problem even if the directions don't ask for a sketch.

GOAL 2 BISECTING AN ANGLE

An **angle bisector** is a ray that divides an angle into two adjacent angles that are congruent. In the diagram at the right, the ray $\overrightarrow{CD}$ bisects $\angle ABC$ because it divides the angle into two congruent angles, $\angle ACD$ and $\angle BCD$.

In this book, matching *congruence arcs* identify congruent angles in diagrams.

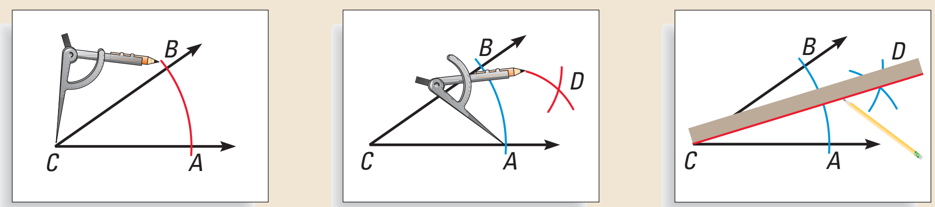


$m\angle ACD = m\angle BCD$

ACTIVITY
Construction

Angle Bisector

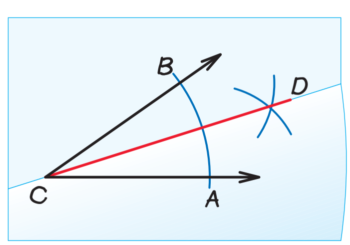
Use the following steps to construct an angle bisector of $\angle C$.



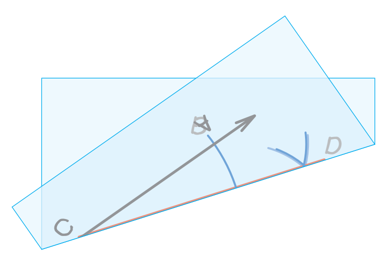
1. Place the compass point at C. Draw an arc that intersects both sides of the angle. Label the intersections A and B.
2. Place the compass point at A. Draw an arc. Then place the compass point at B. Using the same compass setting, draw another arc.
3. Label the intersection D. Use a straightedge to draw a ray through C and D. This is the angle bisector.

After you have constructed an angle bisector, you should check that it divides the original angle into two congruent angles. One way to do this is to use a protractor to check that the angles have the same measure.

Another way is to fold the piece of paper along the angle bisector. When you hold the paper up to a light, you should be able to see that the sides of the two angles line up, which implies that the angles are congruent.



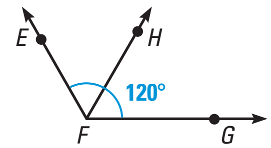
Fold on $\overrightarrow{CD}$.



The sides of angles $\angle BCD$ and $\angle ACD$ line up.

EXAMPLE 3 *Dividing an Angle Measure in Half*

The ray $\overrightarrow{FH}$ bisects the angle $\angle EFG$. Given that $m\angle EFG = 120°$, what are the measures of $\angle EFH$ and $\angle HFG$?



SOLUTION

An angle bisector divides an angle into two congruent angles, each of which has half the measure of the original angle. So,

$$m\angle EFH = m\angle HFG = \frac{120°}{2} = 60°.$$

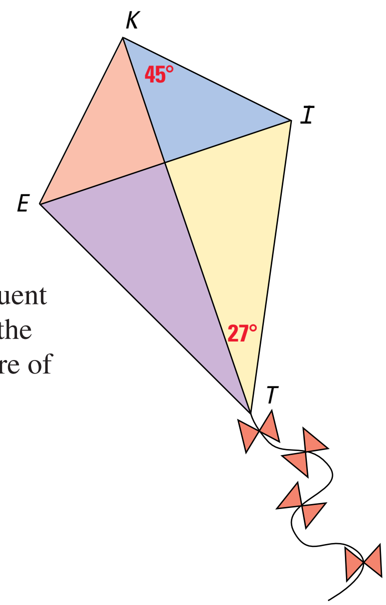
EXAMPLE 4 *Doubling an Angle Measure*

KITE DESIGN In the kite, two angles are bisected.

$\angle EKI$ is bisected by $\overrightarrow{KT}$.

$\angle ITE$ is bisected by $\overrightarrow{TK}$.

Find the measures of the two angles.



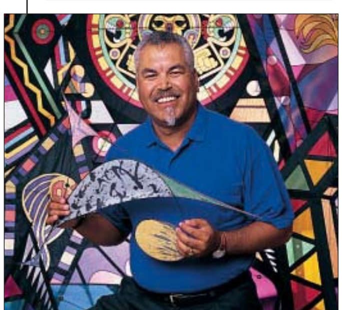
SOLUTION

You are given the measure of one of the two congruent angles that make up the larger angle. You can find the measure of the larger angle by doubling the measure of the smaller angle.

$$m\angle EKI = 2m\angle TKI = 2(45°) = 90°$$

$$m\angle ITE = 2m\angle KTI = 2(27°) = 54°$$

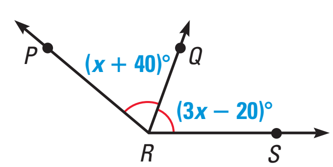
FOCUS ON PEOPLE



REAL LIFE **JOSÉ SAÍNZ,** a San Diego kite designer, uses colorful patterns in his kites. The struts of his kites often bisect the angles they support.

EXAMPLE 5 *Finding the Measure of an Angle*

In the diagram, $\overrightarrow{RQ}$ bisects $\angle PRS$. The measures of the two congruent angles are $(x + 40)°$ and $(3x - 20)°$. Solve for x.



Using Algebra

SOLUTION

$m\angle PRQ = m\angle QRS$	Congruent angles have equal measures.
$(x + 40)° = (3x - 20)°$	Substitute given measures.
$x + 60 = 3x$	Add 20° to each side.
$60 = 2x$	Subtract x from each side.
$30 = x$	Divide each side by 2.

▶ So, $x = 30$. You can check by substituting to see that each of the congruent angles has a measure of 70°.

GUIDED PRACTICE

Vocabulary Check ✓

1. What kind of geometric figure is an *angle bisector*?

Concept Check ✓

2. How do you indicate congruent segments in a diagram? How do you indicate congruent angles in a diagram?

3. What is the simplified form of the Midpoint Formula if one of the endpoints of a segment is (0, 0) and the other is (x, y)?

Skill Check ✓

Find the coordinates of the midpoint of a segment with the given endpoints.

4. $A(5, 4)$, $B(-3, 2)$ **5.** $A(-1, -9)$, $B(11, -5)$ **6.** $A(6, -4)$, $B(1, 8)$

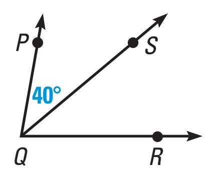
Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint *M*.

7. $C(3, 0)$, $M(3, 4)$ **8.** $D(5, 2)$, $M(7, 6)$ **9.** $E(-4, 2)$, $M(-3, -2)$

10. Suppose $m\angle JKL$ is 90°. If the ray $\overrightarrow{KM}$ bisects $\angle JKL$, what are the measures of $\angle JKM$ and $\angle LKM$?

$\overrightarrow{QS}$ is the angle bisector of $\angle PQR$. Find the two angle measures not given in the diagram.

11.



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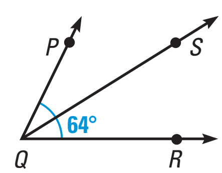

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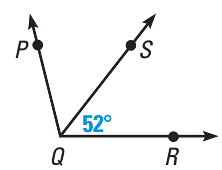


PRACTICE AND APPLICATIONS

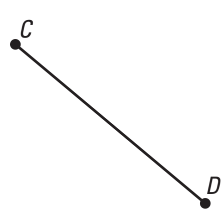
STUDENT HELP

Extra Practice to help you master skills is on p. 804.

CONSTRUCTION **Use a ruler to measure and redraw the line segment on a piece of paper. Then use construction tools to construct a segment bisector.**

14.

15.



16.

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 17–24
Example 2: Exs. 25–30
Example 3: Exs. 37–42
Example 4: Exs. 37–42
Example 5: Exs. 44–49

FINDING THE MIDPOINT **Find the coordinates of the midpoint of a segment with the given endpoints.**

17. $A(0, 0)$, $B(-8, 6)$ **18.** $J(-1, 7)$, $K(3, -3)$ **19.** $C(10, 8)$, $D(-2, 5)$ **20.** $P(-12, -9)$, $Q(2, 10)$

21. $S(0, -8)$, $T(-6, 14)$ **22.** $E(4, 4)$, $F(4, -18)$ **23.** $V(-1.5, 8)$, $W(0.25, -1)$ **24.** $G(-5.5, -6.1)$, $H(-0.5, 9.1)$

USING ALGEBRA Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint M.

25. $R(2, 6)$
$M(-1, 1)$

26. $T(-8, -1)$
$M(0, 3)$

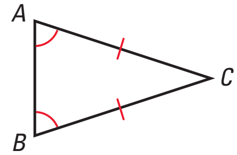
27. $W(3, -12)$
$M(2, -1)$

28. $Q(-5, 9)$
$M(-8, -2)$

29. $A(6, 7)$
$M(10, -7)$

30. $D(-3.5, -6)$
$M(1.5, 4.5)$

RECOGNIZING CONGRUENCE Use the marks on the diagram to name the congruent segments and congruent angles.

31.


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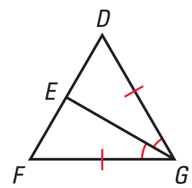

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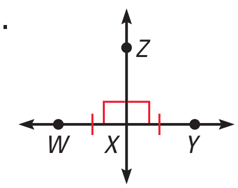

CONSTRUCTION Use a protractor to measure and redraw the angle on a piece of paper. Then use construction tools to find the angle bisector.

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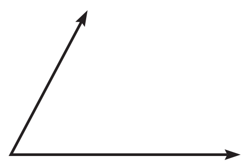

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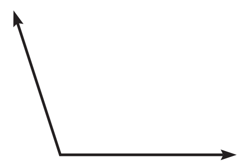

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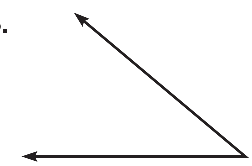
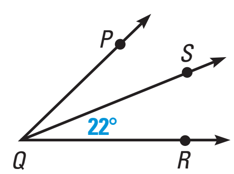


ANALYZING ANGLE BISECTORS $\overrightarrow{QS}$ is the angle bisector of $\angle PQR$. Find the two angle measures not given in the diagram.

37.


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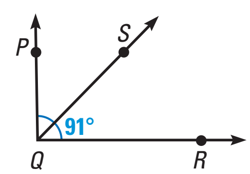

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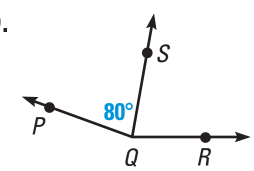

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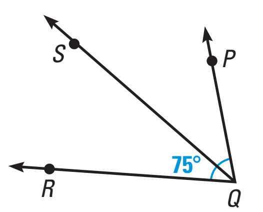

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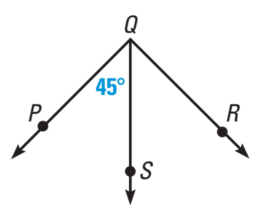

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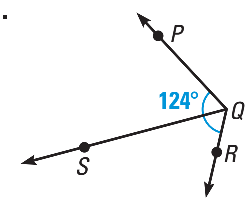
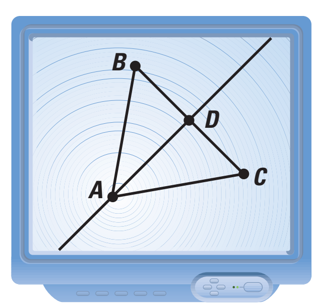


STUDENT HELP

SOFTWARE HELP
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

43. **TECHNOLOGY** Use geometry software to draw a triangle. Construct the angle bisector of one angle. Then find the midpoint of the opposite side of the triangle. Change your triangle and observe what happens.



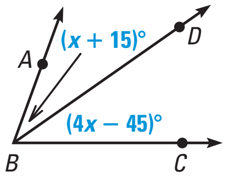
Does the angle bisector *always* pass through the midpoint of the opposite side? Does it *ever* pass through the midpoint?

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Ex. 44–49.

USING ALGEBRA $\overrightarrow{BD}$ **bisects** $\angle ABC$**. Find the value of** x**.**

44.



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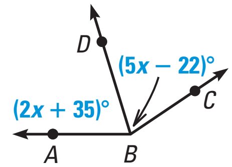

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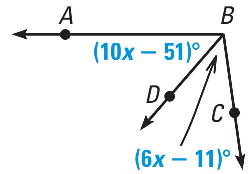

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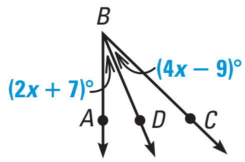

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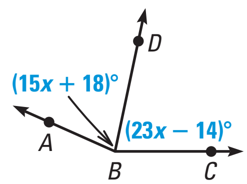

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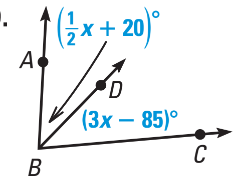
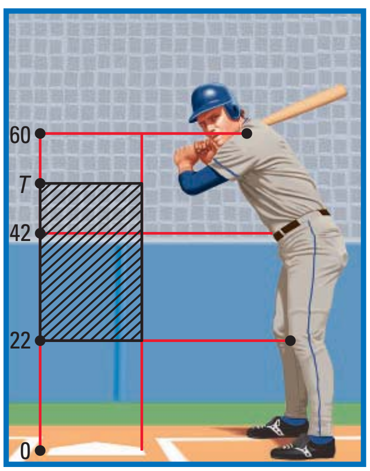


STRIKE ZONE **In Exercises 50 and 51, use the information below. For each player, find the coordinate of** T**, a point on the top of the strike zone.** In baseball, the "strike zone" is the region a baseball needs to pass through in order for an umpire to declare it a strike if it is not hit. The *top of the strike zone* is a horizontal plane passing through the midpoint between the top of the hitter's shoulders and the top of the uniform pants when the player is in a batting stance.

▶ Source: Major League Baseball

50.



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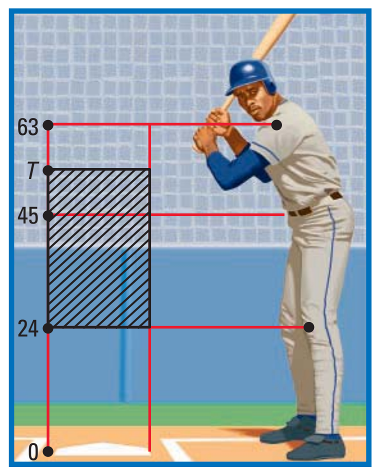
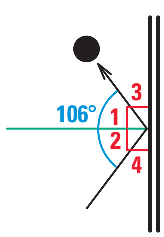


AIR HOCKEY **When an air hockey puck is hit into the sideboards, it bounces off so that** $\angle 1$ **and** $\angle 2$ **are congruent. Find** $m\angle 1$**,** $m\angle 2$**,** $m\angle 3$**, and** $m\angle 4$**.**

52.



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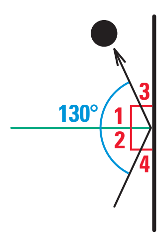

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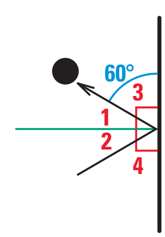
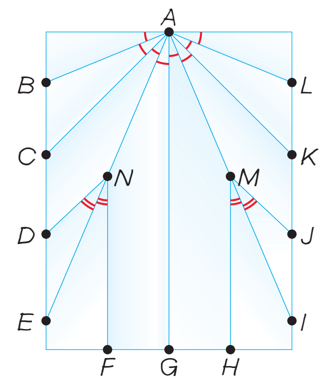


55. **PAPER AIRPLANES** The diagram represents an unfolded piece of paper used to make a paper airplane. The segments represent where the paper was folded to make the airplane.

Using the diagram, name as many pairs of congruent segments and as many congruent angles as you can.



56. *Writing* Explain, in your own words, how you would divide a line segment into four congruent segments using a compass and straightedge. Then explain how you could do it using the Midpoint Formula.

57. **MIDPOINT FORMULA REVISITED** Another version of the Midpoint Formula, for $A(x_1, y_1)$ and $B(x_2, y_2)$, is

$$M\left[x_1 + \frac{1}{2}(x_2 - x_1), y_1 + \frac{1}{2}(y_2 - y_1)\right].$$

Redo Exercises 17–24 using this version of the Midpoint Formula. Do you get the same answers as before? Use algebra to explain why the formula above is equivalent to the one in the lesson.

Test Preparation

58. **MULTI-STEP PROBLEM** Sketch a triangle with three sides of different lengths.

a. Using construction tools, find the midpoints of all three sides and the angle bisectors of all three angles of your triangle.

b. Determine whether or not the angle bisectors pass through the midpoints.

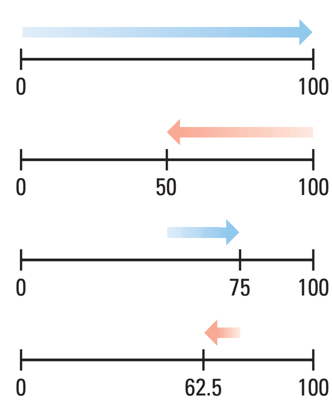
c. *Writing* Write a brief paragraph explaining your results. Determine if your results would be different if you used a different kind of triangle.

★ Challenge

INFINITE SERIES **A football team practices running back and forth on the field in a special way. First they run from one end of the 100 yd field to the other. Then they turn around and run half the previous distance. Then they turn around again and run half the previous distance, and so on.**

59. Suppose the athletes continue the running drill with smaller and smaller distances. What is the coordinate of the point that they approach?

60. What is the total distance that the athletes cover?



EXTRA CHALLENGE
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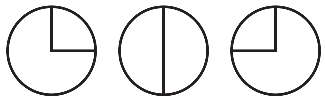
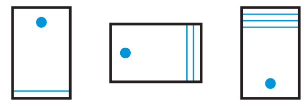
MIXED REVIEW

SKETCHING VISUAL PATTERNS **Sketch the next figure in the pattern.** (Review 1.1)

61. 

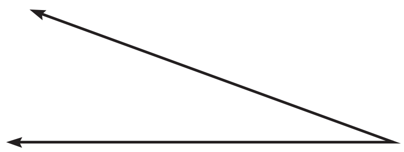
62. 

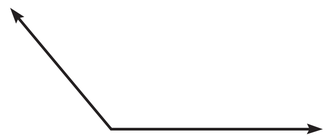
DISTANCE FORMULA **Find the distance between the two points.** (Review 1.3)

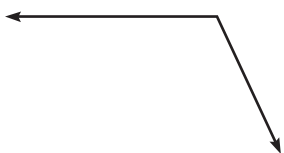
63. $A(3, 12), B(-5, -1)$ **64.** $C(-6, 9), D(-2, -7)$ **65.** $E(8, -8), F(2, 14)$

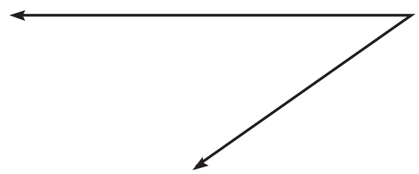
66. $G(3, -8), H(0, -2)$ **67.** $J(-4, -5), K(5, -1)$ **68.** $L(-10, 1), M(-4, 9)$

MEASURING ANGLES **Use a protractor to find the measure of the angle.** (Review 1.4 for 1.6)

69. 

70. 

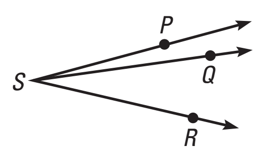
71. 

72. 

QUIZ 2

Self-Test for Lessons 1.4 and 1.5

1. State the Angle Addition Postulate for the three angles shown at the right. (Lesson 1.4)



In a coordinate plane, plot the points and sketch $\angle DEF$. Classify the angle. Write the coordinates of a point that lies in the interior of the angle and the coordinates of a point that lies in the exterior of the angle. (Lesson 1.4)

2. $D(-2, 3)$ $E(4, -3)$ $F(2, 6)$

3. $D(-6, -3)$ $E(0, -5)$ $F(8, -5)$

4. $D(-1, 8)$ $E(-4, 0)$ $F(4, 0)$

5. $D(1, 10)$ $E(1, 1)$ $F(8, 1)$

6. In the diagram, $\overrightarrow{KM}$ is the angle bisector of $\angle JKL$. Find $m\angle MKL$ and $m\angle JKL$. (Lesson 1.5)